Personagens e Datas Importantes na história da Mecânica dos Fluidos

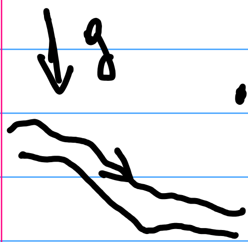
- Arquimedes (308-215 a.C) ⇝ hidrostática

  ⋮ } Idade média

- Leonardo Da Vinci (1452-1519) ⇝ eq. da conservação da massa 1D

  ↳ Descrição cinemática de escoamentos

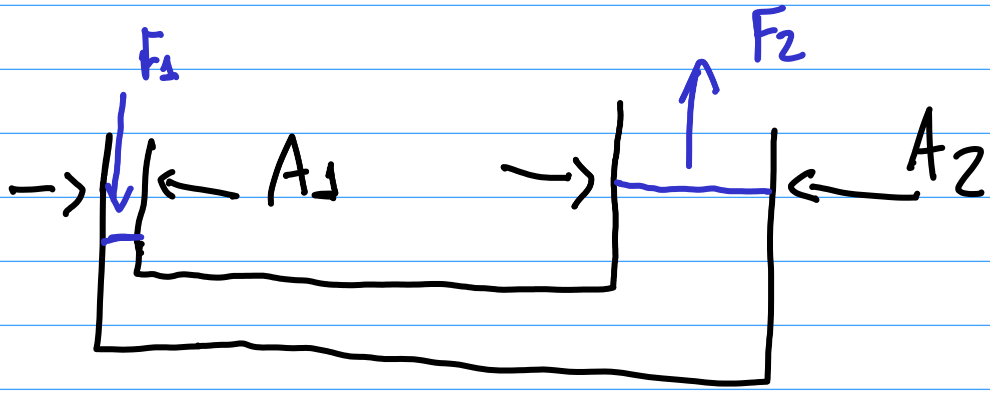
Séculos XVII e XVIII



- Galileu (1564-1642) ⇝ Ideias preliminares sobre gravidade

  } $v \sim \sqrt{gh}$

• Pascal (1623-1662) ⇝ "Treatise on the equilibrium of Liquids"



$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\boxed{F_2 = F_1 \left(\frac{A_2}{A_1}\right)}$$

• Newton (1642-1727)
- ↳ Leis do movimento
- ↳ Teoria da Gravidade (Preliminar)
- ↳ Lei da viscosidade de Newton

- Daniel Bernoulli (1700-1782)
  - ↳ eq de Bernoulli
  - ↳ relações entre pressão e velocidade

$\underbrace{\text{pressão}}_{\text{dinâmica}}$ e $\underbrace{\text{velocidade}}_{\text{cinemática}}$

- Euler (1707 - 1783)
  - ↳ eq. diferencial do movimento p/ fluidos Invíscidos

Século XIX

- Navier (1785-1836) · Stokes (1819-1903)

↓

Tensor de Tensões de Navier-Stokes

Da aula passada:

- Eq. de Cauchy $\rightsquigarrow$ $\rho \dfrac{D\underset{\sim}{v}}{Dt} = \nabla \cdot \underline{\underline{\sigma}} + \rho \underset{\sim}{g}$

- $\underbrace{\text{Princípio de Balanço de momento angular}}$

  ↳ $\underbrace{\text{"Na ausência de torques internos (intrínsecos ao material) o tensor de tensões é simétrico."}}$

$$\boxed{\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T}$$

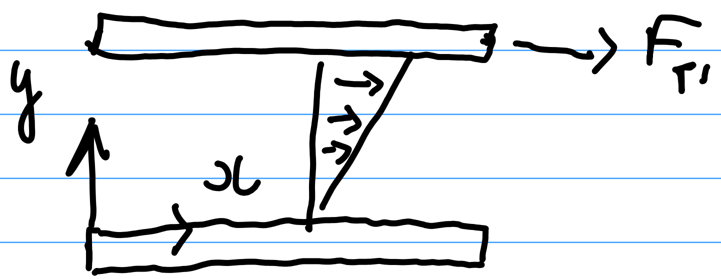
- Isotropia $\rightsquigarrow$ "Um tensor isotrópico é aquele cujo os componentes permanecem inalterados após qualquer transformação ortogonal."

- <u>Representações gerais de tensores isotrópicos</u>
  - ↳ 2ª ordem: $\underline{\underline{A}} = \lambda(\underset{\sim}{x}, t)\,\underline{\underline{I}} \rightsquigarrow A_{ij} = \lambda\,\delta_{ij}$
  - ↳ 4ª ordem $\rightsquigarrow$ <u>Teorema de Cauchy-Weil</u>:

$$\boxed{A_{ijkl} = \alpha\,\delta_{ij}\delta_{kl} + \beta\,\delta_{ik}\delta_{jl} + \lambda\,\delta_{il}\delta_{jk}}$$

Navier-Stokes $\rightsquigarrow$ eq. constitutiva $\rightsquigarrow$ semi-empírico

Ponto de partida ⇝ Lei da Viscosidade de Newton



F_T, τ ↳ $\tau_{yx} = \frac{F_T}{A_p} \propto \frac{dv_x}{dy}$

↳ placa parada

$$\underbrace{\qquad\qquad}$$

$\tau_{yx} = \mu \frac{dv_x}{dy}$

↳ viscosidade

- 1 dos 9 componentes do tensor $\underline{\underline{\tau}}$
- Linkado com 1 dos 9 componentes do tensor $\nabla \underset{\sim}{v}$

Inspiração ⇝ $\underline{\underline{\tau}} \propto \nabla \underset{\sim}{v}$

↳ vem da Lei da viscosidade de Newton

Qual a forma tensorial mais geral possível de se conectar linearmente esses dois tensores?

↳ $\underline{\underline{\tau}} = \mu \nabla \underset{\sim}{v}$; $\underset{\sim}{\mu} \cdot \nabla \underset{\sim}{v} \rightarrow$ vetor ✗

↳ escalar

$\underline{\underline{\mu}} \cdot \nabla \underset{\sim}{v} \rightarrow$ tensor de 2ª ordem ✓

$\mu_{ab} \hat{e}_a \hat{e}_b \cdot \frac{\partial v_d}{\partial x_c} \hat{e}_c \hat{e}_d$

$\leadsto \mu_{ab} \frac{\partial v_d}{\partial x_c} \hat{e}_a \underbrace{(\hat{e}_b \cdot \hat{e}_c)}_{\delta_{bc}} \hat{e}_d = \mu_{ab} \frac{\partial v_d}{\partial x_b} \hat{e}_a \hat{e}_d$

- Uma viscosidade expressa por um tensor viscosidade de 2ª ordem mantém a consistência tensorial da ligação entre $\underline{\underline{\tau}}$ e $\nabla \underset{\sim}{v}$. Mas essa é a forma mais geral possível?

Suponha: $\underline{\underline{\underline{\mu}}} \cdot \nabla \underset{\sim}{v} \rightsquigarrow \underbrace{\mu_{abc}\, \hat{e}_a \hat{e}_b \hat{e}_c \cdot \frac{\partial v_e}{\partial x_d} \hat{e}_d \hat{e}_e}_{\mu_{abc} \frac{\partial v_e}{\partial x_c} \hat{e}_a \hat{e}_b \hat{e}_e \rightsquigarrow 3^{\text{a}} \text{ ordem}}$

ou

$$\underbrace{\underline{\underline{\underline{\mu}}} : \nabla \underset{\sim}{v} \rightarrow ?}_{\mu_{abc}\, \hat{e}_a \hat{e}_b \hat{e}_c : \frac{\partial v_e}{\partial x_d} \hat{e}_d \hat{e}_e}$$

$$\mu_{abc}\, \hat{e}_a \hat{e}_b \hat{e}_c : \frac{\partial v_e}{\partial x_d} \hat{e}_d \hat{e}_e \Big\} \; \mu_{abc} \frac{\partial v_e}{\partial x_d} (\delta_{bd}\, \delta_{ce})\, \hat{e}_a$$

$\hat{e}_a \swarrow 1^{\text{a}}$ ordem

$$e : \quad \underline{\underline{\mu}} : \nabla \underset{\sim}{v} \to \; ? \; \to \mu_{abcd}\, \hat{e}_a \hat{e}_b \hat{e}_c \hat{e}_d : \frac{\partial v_f}{\partial x_e} \hat{e}_e \hat{e}_f$$

$$\mu_{abcd} \frac{\partial v_f}{\partial x_e} \hat{e}_a \hat{e}_b \underbrace{\left(\hat{e}_c \hat{e}_d : \hat{e}_e \hat{e}_f\right)}_{\delta_{ce}\,\delta_{df}} \to \mu_{abcd} \frac{\partial v_d}{\partial x_c} = \tau_{ab}$$

Por enquanto temos: $\tau_{ab} = \mu_{abcd} \dfrac{\partial v_d}{\partial x_c}$ (1)

- <u>Hipótese 1</u>

  ↳ material isotrópico $\to$ μ_{abcd} é isotrópico :

Cauchy-Weil $\rightsquigarrow$ $\mu_{abcd} = \alpha\, \delta_{ab}\delta_{cd} + \beta\, \delta_{ac}\delta_{bd} + \lambda\, \delta_{ad}\delta_{bc}$ (2)

$\alpha, \beta, \lambda \rightsquigarrow$ escalares | Isotropia

$\hookrightarrow$ 81 viscosidades

$\hookrightarrow$ 3

$$(2) \to (1): \quad \underbrace{\tau_{ab} = \alpha\, \delta_{ab}\delta_{cd} \frac{\partial v_d}{\partial x_c} + \beta\, \delta_{ac}\delta_{bd} \frac{\partial v_d}{\partial x_c} + \lambda\, \delta_{ad}\delta_{bc} \frac{\partial v_d}{\partial x_c}}$$

$$\boxed{\tau_{ab} = \alpha \frac{\partial v_c}{\partial x_c} \delta_{ab} + \beta \frac{\partial v_b}{\partial x_a} + \lambda \frac{\partial v_a}{\partial x_b}} \quad (3)$$

- Hipótese 2 $\to$ $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T \rightsquigarrow \underline{\underline{\tau}} = \underline{\underline{\tau}}^T \rightsquigarrow \tau_{ab} = \tau_{ba}$

$$\underbrace{\alpha \cancel{\frac{\partial v_c}{\partial x_c}} \delta_{ab} + \beta \frac{\partial v_b}{\partial x_a} + \lambda \frac{\partial v_a}{\partial x_b}}_{\tau_{ab}} = \underbrace{\alpha \cancel{\frac{\partial v_c}{\partial x_c}} \delta_{ba} + \beta \frac{\partial v_a}{\partial x_b} + \lambda \frac{\partial v_b}{\partial x_a}}_{\tau_{ba}}$$

$$\beta \left(\frac{\partial v_b}{\partial x_a} - \cancel{\frac{\partial v_a}{\partial x_b}} \right) = \lambda \left(\frac{\partial v_b}{\partial x_a} - \cancel{\frac{\partial v_a}{\partial x_b}} \right) \rightarrow \boxed{\beta = \lambda} \quad (4)$$

- Simetria $\rightarrow$ 3 constantes materiais $\rightarrow$ 2

$(4) \rightarrow (3)$:

$$\tau_{ab} = \alpha \frac{\partial v_c}{\partial x_c} \delta_{ab} + \beta \left(\frac{\partial v_a}{\partial x_b} + \frac{\partial v_b}{\partial x_a} \right) \quad (5)$$

$$\underline{\underline{\sigma}} = -p\,\underline{\underline{I}} + \underline{\underline{\tau}}$$

$p \rightarrow$ termodinâmica

$\hookrightarrow$ que pressão é essa?

$$\boxed{P_{\text{mecânica}} = -\frac{1}{3}\,\mathrm{tr}\left(\underline{\underline{\sigma}}\right)}$$

Hipótese de fluido Stokesiano $\rightsquigarrow$ $\boxed{P = P_{\text{mecânica}}}$

Aplicando essa hipótese: $\sigma_{ab} = -p\,\delta_{ab} + \alpha \dfrac{\partial v_c}{\partial x_c}\delta_{ab} + \beta\left(\dfrac{\partial v_a}{\partial x_b} + \dfrac{\partial v_b}{\partial x_a}\right)$

$$-\frac{\sigma_{aa}}{3} = P = p\frac{\delta_{aa}}{3} - \alpha\frac{\partial v_c}{\partial x_c}\frac{\delta_{aa}}{3} - \frac{\beta}{3}\left(\frac{\partial v_a}{\partial x_a} + \frac{\partial v_a}{\partial x_a}\right)$$

$$\underbrace{P = p - \alpha \nabla \cdot \underset{\sim}{v} - \frac{2\beta}{3} \nabla \cdot \underset{\sim}{v}} \rightsquigarrow \boxed{\alpha = -\frac{2}{3}\beta}$$

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \beta\left[\nabla \underset{\sim}{v} + (\nabla \underset{\sim}{v})^T\right] - \frac{2}{3}\beta(\nabla \cdot \underset{\sim}{v})\underline{\underline{I}}$$

chamando $\beta = \mu \rightarrow$ viscosidade:

tensor de tensões de Navier-Stokes

$$\boxed{\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \mu\left[\nabla \underset{\sim}{v} + (\nabla \underset{\sim}{v})^T\right] - \frac{2}{3}\mu(\nabla \cdot \underset{\sim}{v})\underline{\underline{I}}}$$