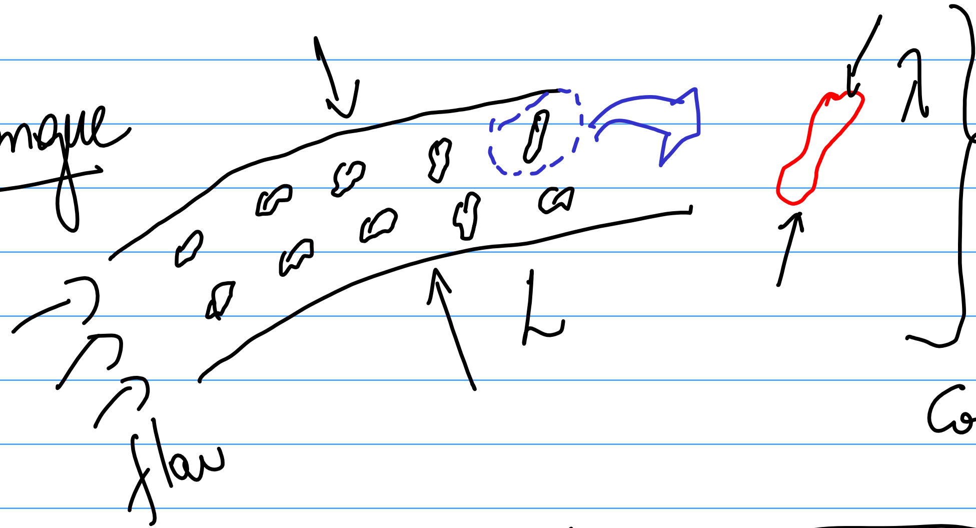
Nusselt → $Nu = \dfrac{hL}{k}$

Knudsen → $Kn = \dfrac{\lambda}{L}$ λ → escala interna, L → escala externa

Sangue



$\lambda << L$ → $Kn << 1$ → Contínuo

Da última aula: $$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}_{\text{Convecção}} = \underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Difusão}} \qquad (1)$$

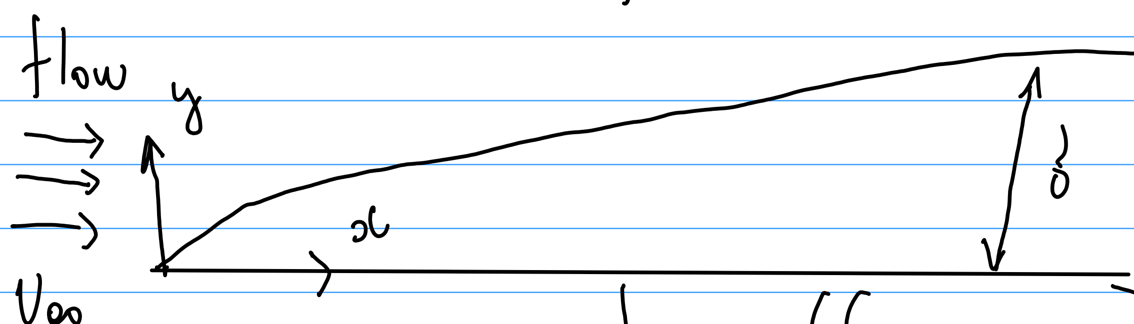
C.L. térmica: $$\overbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}} = \overbrace{\alpha\frac{\partial^2 T}{\partial y^2}}$$

C.L concentração: $$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = \mathcal{D}_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

Análise de escala bem simples em cima das eq. de Prandtl:

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}_{\frac{U_\infty^2}{L} \sim \nu\frac{U_\infty}{\delta^2}} \;\rightarrow\; \underbrace{\frac{\delta^2}{L^2} \sim \frac{\nu \cancel{L}}{U_\infty} \times \frac{1}{L^{\cancel{2}}}}_{\frac{\delta}{L} \sim \left(\frac{\nu}{U_\infty L}\right)^{1/2}}$$

$$\boxed{\frac{\delta}{L} \sim Re_L^{-1/2}}\;(*) \;\rightsquigarrow\; \boxed{\frac{\delta(x)}{x} \sim Re_x^{-1/2}}$$

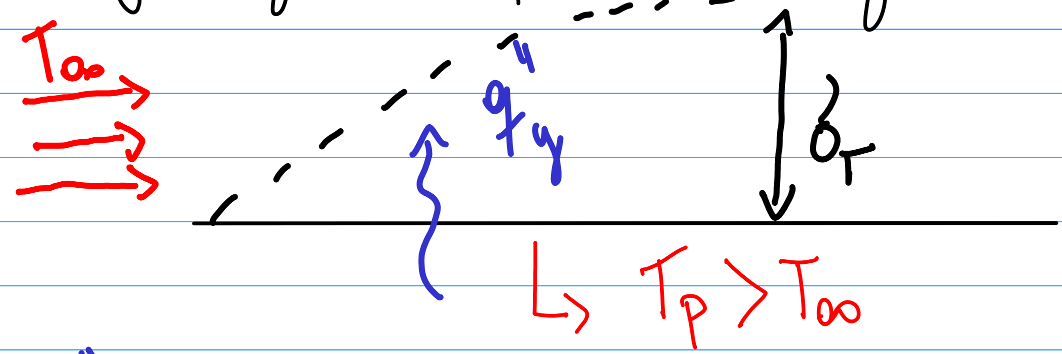


$$F_D = ? \;\rightsquigarrow\; \boxed{F_D = \iint_{z\;x} \tau_{yx}\Big|_{y=0} dx\,dz}$$

$$\boxed{\tau_{yx}\Big|_{y=0} = \mu \frac{\partial u}{\partial y}\Big|_{y=0}} \quad (1)$$

↳ $\tau_{yx}|_{y=0} = \tau_w$

Imagine que esta placa está aquecida:



$$q''_y = h\,(T_p - T_\infty) = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Definimos: $$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{\mu \left.\frac{du}{dy}\right|_{y=0}}{\frac{1}{2}\rho U_\infty^2}$$

$\swarrow$ Coef. de atrito

$$C_f \sim \frac{\mu}{\rho}\,\frac{U_\infty}{U_\infty x}\,\frac{1}{\delta} \sim \frac{Re_L^{\frac{1}{2}}}{L}\,\frac{\mu}{U_\infty\,\rho}$$

$$C_f \sim \frac{Re_L^{1/2}}{Re_L} \rightarrow \boxed{C_f \sim Re_L^{-\frac{1}{2}}}$$

Da eq. da energia: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$\frac{U_\infty \cancel{\Delta T}}{L} \sim \alpha \frac{\cancel{\Delta T}}{\delta_T^2} \rightarrow \frac{\delta_T^2}{L^2} \sim \frac{\alpha \cancel{L}}{U_\infty} \frac{1}{L^{\cancel{2}}}$$

$$\frac{\delta_T}{L} \sim \left(\frac{\alpha}{U_\infty L} \times \frac{\nu}{\nu} \right)^{\frac{1}{2}}$$

$$\frac{1}{Re_L} \quad \frac{1}{Pr}$$

$$\boxed{\frac{\delta_T}{L} \sim (Re_L Pr)^{-\frac{1}{2}}}$$

$$Nu = \frac{hL}{k} \sim \frac{\cancel{k}}{\delta_T} \frac{L}{\cancel{k}}$$

Balanço de energia: $h \cancel{\Delta T} \sim k \frac{\cancel{\Delta T}}{\delta_T}$

$$Nu \sim \frac{1}{\delta_T} \rightarrow \boxed{Nu \sim Re_L^{\frac{1}{2}} Pr^{\frac{1}{2}}}$$

Soluções Integrais $\rightarrow$ Karman - Pohlhausen:

Note que: $\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) = u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x}$

$$+ u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}$$

$$\frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + u\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)}_{\nabla \cdot \underset{\sim}{v} = 0} \rightarrow \text{Incompressível}$$

Eq. de Prandtl $\rightsquigarrow$ $$\frac{\partial u^2}{\partial x} + \frac{\partial}{\partial y}(uv) = \nu \frac{\partial^2 u}{\partial y^2}$$

Integrando:

(2) $$\int_0^\infty \frac{\partial u^2}{\partial x}\,dy + \underbrace{(uv)\Big|_0^\infty}_{U_\infty V_\infty - U_0 V_0} = \nu\left(\frac{\partial u}{\partial y}\Big|_\infty - \frac{\partial u}{\partial y}\Big|_0\right)$$

$U_0 = V_0 = 0$; $\dfrac{\partial u}{\partial y}\Big|_\infty = 0$; Note que: $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

$$\int_0^\infty \frac{\partial u}{\partial x}\,dy + V_\infty - \cancelto{0}{V_0} = 0$$

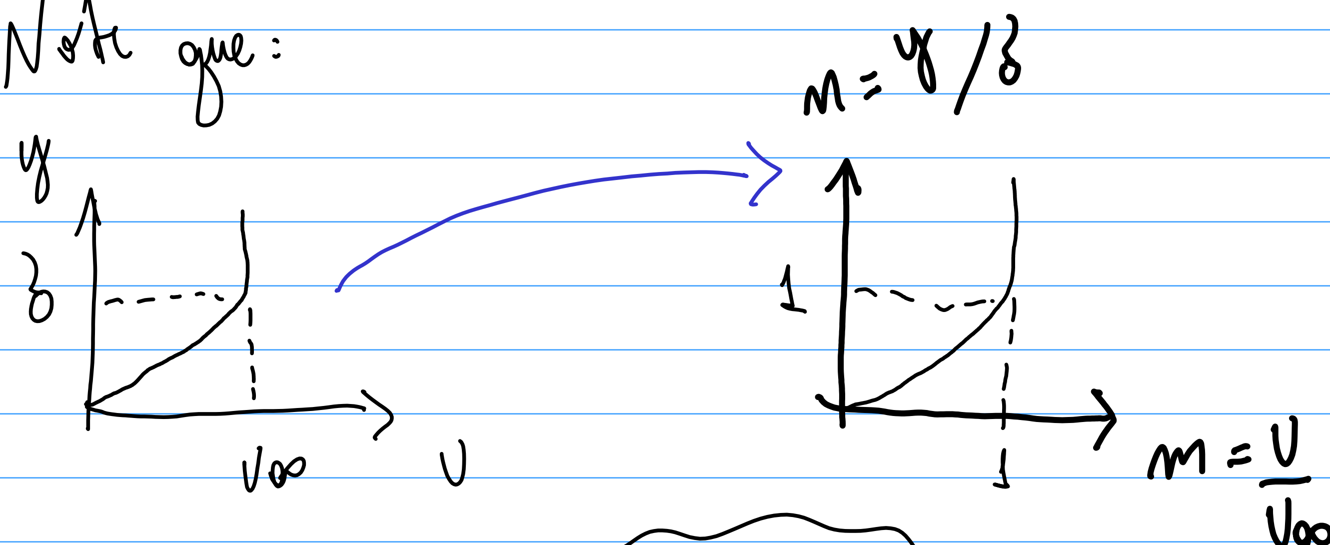
$$\boxed{V_\infty = -\int_0^\infty \frac{\partial u}{\partial x}\,dy} \qquad (3)$$

(3) → (2):

$$\int_0^\infty \frac{\partial U^2}{\partial x}\, dy - U_\infty \int_0^\infty \frac{\partial U}{\partial x}\, dy = -\nu \frac{\partial U}{\partial y}\Big|_{y=0}$$

(4)
$$\boxed{\int_0^\infty \frac{\partial}{\partial x}\left[U\left(U - U_\infty\right)\right] dy = -\nu \frac{\partial U}{\partial y}\Big|_{y=0}}$$

Note que:



$$\boxed{dy = \delta\, d\eta \; ; \; U = m\, U_\infty \;\; \hookrightarrow m(\eta)} \quad (5)$$

(4) → (5):

$$U_\infty^2 \int_0^1 \frac{\partial}{\partial x}\left[m\,(m-1)\right] \delta\, d\eta = -\frac{\nu U_\infty}{\delta} \frac{dm}{d\eta}\Big|_{\eta=0}$$

$$\delta \frac{d\delta}{dx} \underbrace{\int_0^1 m(m-1)\,dn}_{A} = -\frac{\nu}{U_\infty} \underbrace{\frac{dm}{dn}\Big|_{n=0}}_{B}$$

$$\int_0^{\delta(x)} \delta \, d\delta = -\frac{\nu}{U_\infty}\left(\frac{B}{A}\right)\int_0^x dx$$

$$\frac{\delta^2}{2} = -\frac{\nu x}{U_\infty}\left(\frac{B}{A}\right) \quad \div \text{ por } x^2 :$$

$$\left(\frac{\delta}{x}\right)^2 = -\frac{2B}{A}\underbrace{\left(\frac{\nu}{U_\infty x}\right)}_{Re_x^{-1}} \rightsquigarrow \frac{\delta}{x} = \left(\frac{-2B}{A}\right) Re_x^{-\frac{1}{2}}$$

Na era Moderna ⇝ CFD ⇝ Dynamics

Computational

Fluid

CFD
1 - DNS ⇝ Direct Numerical Simulation
2 - LES ⇝ Large Eddy Simulation
3 - RANS ⇝ Reynolds Average Navier-Stokes

$$\boxed{N \sim Re^{9/4}}$$

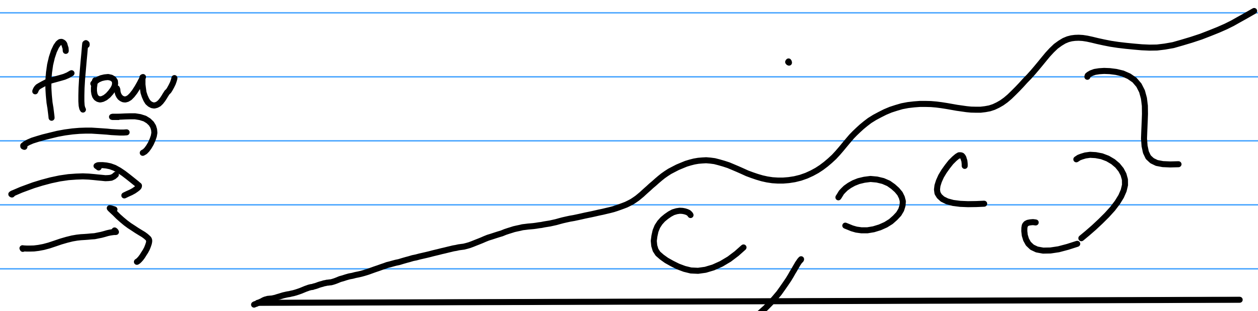
Reynolds Industrial típico ⇝ $Re \sim 10^6$

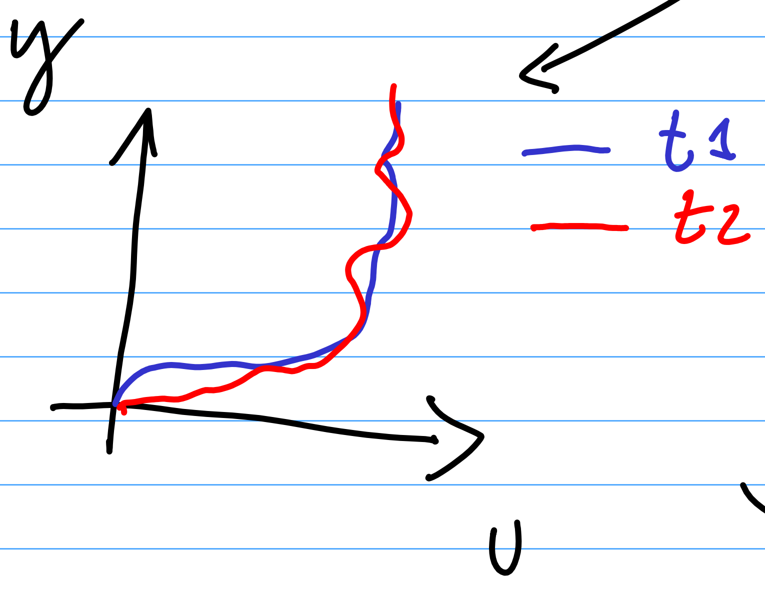
$$\boxed{N \sim 10^{14}}$$

Rans

↳ Decomposição de Reynols

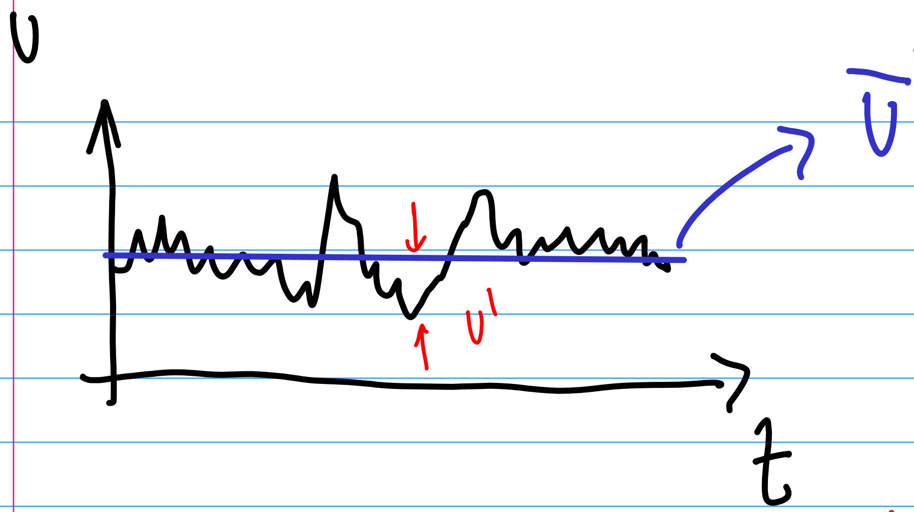
Camada Limite Turbulenta





Reynolds em 1895 propõe o seguinte tratamento estatístico:

Considere: $\bar{U} = \frac{1}{T}\int_0^T U(t)\,dt$; $\bar{V} = \frac{1}{T}\int_0^T V(t)\,dt$



$$u(\underset{\sim}{x},t) = \bar{u}(\underset{\sim}{x}) + u'(\underset{\sim}{x},t)$$

$$v(\underset{\sim}{x},t) = \bar{v}(\underset{\sim}{x}) + v'(\underset{\sim}{x},t)$$

Decomposição de Reynolds

Considere as seguintes propriedades:

- $\overline{u'} = \overline{v'} = 0$
- $\overline{u+v} = \bar{u} + \bar{v}$
- $\overline{\bar{u}\,u'} = 0$
- $\overline{uv} = \bar{u}\,\bar{v} + \overline{u'v'}$
- $\overline{u^2} = \bar{u}^2 + \overline{u'^2}$
- $\overline{\dfrac{\partial u}{\partial x}} = \dfrac{\partial \bar{u}}{\partial x}$
- $\overline{\dfrac{\partial u}{\partial t}} = \dfrac{\partial \bar{u}}{\partial t} = 0$

Eq. de Prandtl: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7)$$

$$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \qquad (8)}$$

$$u = \bar{u} + u' \quad ; \quad v = \bar{v} + v' \qquad (9)$$

(9) → (7): $$\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial v'}{\partial y} = 0 \qquad (10)$$

Aplicando média temporal em (10):

$$\underbrace{\overline{\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial v'}{\partial y}} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \cancel{\frac{\partial \overline{u'}}{\partial x}}^{\to 0} + \cancel{\frac{\partial \overline{v'}}{\partial y}}^{\to 0} = 0}$$

$$\boxed{\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0} \quad e \quad \boxed{\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0} \qquad (11)$$

Aplicando o mesmo procedimento para (8):

$$(\bar{u}+u')\left[\frac{\partial \bar{u}}{\partial x}+\frac{\partial u'}{\partial x}\right]+(\bar{v}+v')\left[\frac{\partial \bar{u}}{\partial y}+\frac{\partial u'}{\partial y}\right]=\nu\left(\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{\partial^2 u'}{\partial y^2}\right)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{u}\frac{\partial u'}{\partial x}+u'\frac{\partial \bar{u}}{\partial x}+u'\frac{\partial u'}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{v}\frac{\partial u'}{\partial y}+v'\frac{\partial \bar{u}}{\partial y}+v'\frac{\partial u'}{\partial y}=$$

$$\nu\left(\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{\partial^2 u'}{\partial y^2}\right)$$

$$\left(\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}\right)+\bar{u}\frac{\partial u'}{\partial x}+u'\frac{\partial \bar{u}}{\partial x}+u'\frac{\partial u'}{\partial x}+\bar{v}\frac{\partial u'}{\partial y}+v'\frac{\partial \bar{u}}{\partial y}$$

$$+v'\frac{\partial u'}{\partial y}=\nu\frac{\partial^2 \bar{u}}{\partial y^2}+\nu\frac{\partial^2 u'}{\partial y^2} \qquad (12)$$

Note que:

$$\left.\begin{aligned} \bar{V}\frac{\partial u'}{\partial y} &= \frac{\partial}{\partial y}(\bar{V}u') - u'\frac{\partial \bar{V}}{\partial y} \\ &\text{e} \\ \bar{u}\frac{\partial u'}{\partial x} &= \frac{\partial}{\partial x}(u'\bar{u}) - u'\frac{\partial \bar{u}}{\partial x} \end{aligned}\right\} \quad (13)$$

(13) → (12) :

$$\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) + \frac{\partial(\bar{v}u')}{\partial y} + u'\frac{\partial \bar{u}}{\partial x} + u'\frac{\partial u'}{\partial x} + \frac{\partial}{\partial x}(u'\bar{u}) + v'\frac{\partial \bar{u}}{\partial y}$$

$$+ v'\frac{\partial u'}{\partial y} - u'\underbrace{\cancel{\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}\right)}}_{\to 0} = \nu\frac{\partial^2 \bar{u}}{\partial y^2} + \nu\frac{\partial^2 u'}{\partial y^2} \quad (14)$$

Note também que:

$$\frac{\partial}{\partial x}(u'u') = u'\frac{\partial u'}{\partial x} + u'\frac{\partial u'}{\partial x}$$

$$\frac{\partial}{\partial y}(u'v') = u'\frac{\partial v'}{\partial y} + v'\frac{\partial u'}{\partial y}$$

$$\underbrace{\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) + \frac{\partial(\bar{v}u')}{\partial y} + u'\frac{\partial \bar{u}}{\partial x} + \frac{\partial(u'^2)}{\partial x} + \frac{\partial(u'\bar{u})}{\partial x} + v'\frac{\partial \bar{u}}{\partial y} + \frac{\partial(u'v')}{\partial y} - u'\left(\frac{\partial u'}{\partial x} + \cancel{\frac{\partial v'}{\partial y}}\right) = \nu\frac{\partial^2\bar{u}}{\partial y^2} + \nu\frac{\partial^2 u'}{\partial y^2}}_{0}$$

Organizando...

$$\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) + \left(\frac{\partial}{\partial x}(u'\bar{u}) + \frac{\partial}{\partial y}(u'\bar{v})\right) + \left(u'\frac{\partial \bar{u}}{\partial x} + v'\frac{\partial \bar{u}}{\partial y}\right)$$

$$+ \left(\frac{\partial(u'^2)}{\partial x} + \frac{\partial(u'v')}{\partial y}\right) = \nu\left(\frac{\partial^2\bar{u}}{\partial y^2} + \frac{\partial^2 u'}{\partial y^2}\right) \quad (15)$$

Aplicando a média temporal em (15):

$$\overline{\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right)} + \overline{\left(\cancel{\frac{\partial}{\partial x}(u'\bar{u})} + \cancel{\frac{\partial}{\partial y}(u'\bar{v})}\right)} + \overline{\left(\cancel{u'\frac{\partial \bar{u}}{\partial x}} + \cancel{v'\frac{\partial \bar{u}}{\partial y}}\right)}$$

$$+ \overline{\left(\frac{\partial(u'^2)}{\partial x} + \frac{\partial(u'v')}{\partial y}\right)} = \nu\,\overline{\left(\frac{\partial^2\bar{u}}{\partial y^2} + \cancel{\frac{\partial^2 u'}{\partial y^2}}\right)}$$

Após a aplicação da média:

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \underbrace{\frac{\partial}{\partial x}\left(\overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\overline{u'v'}\right)}_{\text{Termos Extras!!!}} = \nu \frac{\partial^2 \bar{u}}{\partial y^2}$$

↳ Termos Extras!!!

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) + \frac{\partial}{\partial x}\left(\rho \overline{u'^2}\right) + \frac{\partial}{\partial y}\left(\rho \overline{u'v'}\right) = \mu \frac{\partial^2 \bar{u}}{\partial y^2}$$

Definimos aqui o tensor de Reynolds:

$$\tau_{ij}^{T} = \rho\, \overline{u_i' u_j'} = \rho \begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'^2} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'^2} \end{bmatrix}$$

Num contexto turbulento podemos propor algo do tipo:

$$\rho \frac{\bar{D}\bar{\underset{\sim}{v}}}{\bar{D}t} = \nabla \cdot \bar{\underline{\underline{\sigma}}} + \rho \underset{\sim}{g}$$

tensor de Reynolds

$$\bar{\underline{\underline{\sigma}}} = -\bar{p}\,\underline{\underline{I}} + 2\eta \bar{\underline{\underline{D}}} + \underline{\underline{\tau}}^{T}$$

Em 1877 → Boussinesq propôs por analogia:

$$\underline{\underline{\tau}}^{T} = 2\eta_T \bar{\underline{\underline{D}}}$$

viscosidade turbulenta

RANS modula $\mu_T = f(\kappa, \varepsilon)$

$K \to$ energia cinética turbulenta

$\varepsilon \to$ taxa de dissipação da energia cinética turbulenta