

The approximate number of magnetic centres produced by oxygen in dried bacteria is about 2×10^{16} spins/gm. This figure is probably correct only within a factor of two owing to the high gain needed in the detection system of the apparatus with consequent drift and noise when the signals of these materials were determined. Since the number of bacterial cells in 1 gm. of dry material is roughly 4×10^{12} , the number of free radicals produced for each dry cell reacting with oxygen is about 5,000. Possibly the oxygen reaction is quite specific and involves only few molecular species the normal state of which might be vital to the survival of the cell. It also seems that the 'oxygen acceptor' in this reaction is quite heat-sensitive while the reaction product is not.

The experiments of Miyagawa *et al.*⁵ were done on ground, non-viable animal tissues and are therefore not directly comparable with our results. They show that molecular species of biological origin which in the dry state are able to react quickly with oxygen are not restricted to bacteria. Our results differ in that the reaction between oxygen and the sensitive site of bacteria seems to be irreversible.

This last observation would exclude a normal metabolic role for the reaction, for otherwise one

might expect a resonance signal in dried bacteria (which are intact and viable cells) kept in vacuum. One might rather imagine that the site sensitive to oxygen is protected in a wet-cell system by the action of normal metabolic machinery in removing excess oxygen. Once this normal machinery is stopped in the dry cell, the site becomes easily accessible to oxygen. Unless other protecting materials are present, there follow consequent inactivation and loss of the capability of the cell to recover when it is re-introduced into its normal environment. This latter property of the bacterial cell makes it an interesting model for investigating biological phenomena by means of electron-spin resonance.

The Oak Ridge National Laboratory is operated by the Union Carbide Corporation for the U.S. Atomic Energy Commission.

¹ Lion, M. B., and Bergman, E. D., *J. Gen. Microbiol.*, **24**, 191 (1961).

² Conger, A. D., and Randolph, M. L., *Rad. Res.*, **11**, 54 (1959).

³ Kirby-Smith, J. S., and Randolph, M. L., *Intern. J. Rad. Biol. (Suppl.)*, **11** (1960).

⁴ Randolph, M. L., *Rev. Sci. Instr.*, **31**, 949 (1960).

⁵ Miyagawa, I., Gordy, W., Watabe, N., and Wilbur, K. M., *Proc. U.S. Nat. Acad. Sci.*, **44**, 613 (1958).

⁶ Lion, M. B., and Bergman, E. D., *J. Gen. Microbiol.* (in the press).

ANGULAR MOMENTUM OF CONTINUA

By PROF. J. S. DAHLER and PROF. L. E. SCRIVEN

Department of Chemical Engineering, University of Minnesota, Minneapolis

WHEN applied to the continuum model of matter, the principle of conservation of linear momentum becomes Cauchy's law of motion, which in spatial co-ordinates is the familiar equation of change of linear momentum. In Cartesian tensor notation, with D/Dt the substantial time derivative and other symbols having their usual meanings:

$$\rho \frac{D V_j}{D t} = \rho F_j + T_{ij,i} \quad (1)$$

The stress-tensor can be resolved into symmetrical and anti-symmetrical parts $T_{ij}^{(s)} + T_{ij}^{(a)}$; the second is given by $(1/2) e_{ijk} A_k$, where the pseudo-vector $A_k \equiv e_{kmn} T_{mn}$. With this resolution, the first moment of equation (1) about an arbitrarily chosen origin is found to be:

$$\rho \frac{D}{D t} (e_{kij} R_i V_j) = \rho e_{kij} R_i F_j + (e_{kij} R_i T_{ij})_{,l} - A_k \quad (2)$$

This equation of change of moment of momentum, or external angular momentum, $e_{kij} R_i V_j$ (per unit mass), shows that quantity to be conserved in the usual sense only when the stress-tensor happens to be symmetrical; for otherwise A_k does not vanish, and the last term represents local consumption (or production) of moment of momentum throughout the system. What is the fate of this external angular momentum?

A system may exchange with its surroundings not only linear but also angular momentum. The two classes of interaction need not be wholly equivalent in the case of real materials, although they are so regarded in most traditional continuum models. But

just as the distant material surroundings may exert locally a vector body force F_j , they may also exert locally on the system a pseudovector body-couple G_j per unit mass: and just as the contiguous material surroundings may exert a vector surface traction $T_j = n_i T_{ij}$, they may also exert on the surface of the system a pseudovector couple $C_j = n_i C_{ij}$ per unit area, where C_{ij} is the couple stress-tensor. If the pseudovector L_k denotes the local density of angular momentum per unit mass, the principle of conservation of angular momentum for the constant mass system associated with an arbitrary, convected volume is:

$$\frac{d}{dt} \int_V \rho L_k dV = \int_V \rho (e_{kij} R_i F_j + G_k) dV + \int_S n_i (e_{kij} R_i T_{ij} + C_{ik}) dS \quad (3)$$

In the usual way this leads directly to the equation of change of total angular momentum:

$$\rho \frac{D L_k}{D t} = \rho e_{kij} R_i F_j + \rho G_k + (e_{kij} R_i T_{ij})_{,l} + C_{ik,l} \quad (4)$$

Even more important than this equation is the following, obtained by subtracting equation (2) from equation (4) and denoting as M_k the difference $L_k - e_{kij} R_i V_j$ between total angular momentum and moment of momentum:

$$\rho \frac{D M_k}{D t} = \rho G_k + C_{ik,l} + A_k \quad (5)$$

This equation of change of internal angular momentum makes clear the fate of the external angular momen-

tum: it is transmuted to the internal form by the alchemy of an unsymmetrical state of stress. After transformation it may be accumulated within the system or removed to the surroundings. Just as the body-force is the rate of arrival of linear momentum from distant material surroundings, the body-couple G_k is the rate of arrival of internal angular momentum from the same source; and in a similar analogy the net surface couple $C_{ik,j}$ is the rate of arrival, by diffusion, of internal angular momentum from contiguous material surroundings.

So far as we know, equation (5) was first given in complete form in 1959 by Dahler¹, who derived it as a general hydrodynamical equation in the context of a statistical mechanical theory of transport processes in fluids composed of polyatomic molecules. In his developments for classical and quantum cases, patterned after the methods of Irving and Kirkwood and of Irving and Zwanzig, a key step (as in the continuum development here) consists of distinguishing interactions of a system with distant from those with contiguous surroundings. Somewhat earlier Grad² had argued in favour of a version lacking only the body-couple term. Much earlier the Cosserats³ had given another version lacking the accumulation term; long neglected, theirs has been revived in the modern theory of rheology (cf. refs. 4 and 5) and re-discovered in the theory of elasticity⁶. Other incomplete versions have appeared in various contexts, for example, the theories of liquid crystals and of what are called anisotropic fluids^{7,8}.

The derivation sketched here confirms that equation (5) is a basic law of continuum mechanics fully as general as Cauchy's. What are some of its consequences? In the first place, equations (2) and (5) indicate that moment of momentum and internal angular momentum are separately conserved only when the local state of stress is symmetrical. Therefore the usual proof of symmetry fails when there are couple stresses and accumulation of internal angular momentum, as Grad² has observed, as well as when there are body couples to be accounted for. This implies that whenever the adoption of a continuum model requires averaging over the centres of mass of sub-continuum units having rotational degrees of freedom, the stress-tensor is in general asymmetrical, its asymmetry stemming from the interaction of the centres-of-mass system with an interpenetrating twist or spin system¹. An example is provided by Toupin's theory of the elastic dielectric⁴; a farther-reaching one is provided by the new Laval-LeCorre-Raman theory of elasticity^{6,9,10}. Indeed, equation (5) is a means of joining that theory to extensive researches on nuclear and molecular spin systems in the solid

state. It suggests, incidentally, a need for directing attention to the angular momentum aspect of magnetism in quantitative descriptions of macroscopic behaviour of not only solids but also fluids and less-well-defined forms of matter. As yet, in the physics of fluids there are few examples of antisymmetrical states of stress; predicted for polyatomic fluids in general by the statistical mechanical theory, they seem to have been first considered in detail only recently by Ericksen in a theory of certain non-Newtonian fluids^{7,8}. The complete equation of internal angular momentum, which is basic to any full consideration of such states of stress, merits further attention in rheology, particularly as an aid to understanding non-linear behaviour. There might be interesting consequences for the hydrodynamic theory of helium II. The possible existence of asymmetrical stress seems to have been overlooked, and perhaps the peculiar system of spin-excitations in that liquid might be more conveniently described in terms of intrinsic angular momentum density and equation (5) than in terms of vorticity singularities. The equation does suggest the existence of unexplored effects of interchange between moment of momentum and internal angular momentum in macroscopic systems rotated at high angular velocities.

Another consequence of the angular momentum equation is the need it shows for discovering the constitutive relations giving the dependences of anti-symmetrical stress, couple stress, and body-couples on a set of independent variables that completely describes the dynamical state. This need compels one to recognize that continuum models of real matter must be endowed with point structure, a notion that goes back to Duhem and the Cosserats³⁻⁵. This structure must then be characterized quantitatively. Further work on energy and entropy accounting, on constitutive relations, on certain applications to polar fluids, and on consequents for turbulent flow is in progress.

We are grateful to Prof. W. H. Schwartz, of Stanford University, for bringing the most recent papers by Ericksen to our attention.

¹ Dahler, J. S., *J. Chem. Phys.*, **30**, 1447 (1959).

² Grad, H., *Commun. Pure App. Math.*, **5**, 455 (1952).

³ Truesdell, C., *J. Ratl. Mech. Anal.*, **1**, 158 (1952).

⁴ Toupin, R. A., *J. Ratl. Mech. Anal.*, **5**, 806 (1956).

⁵ Ericksen, J. L., and Truesdell, C., *Arch. Ratl. Mech. Anal.*, **1**, 295 (1958).

⁶ Tiffen, R., and Stevenson, A. C., *Quart. J. Mech. App. Math.*, **9**, 306 (1956).

⁷ Ericksen, J. L., *Arch. Ratl. Mech. Anal.*, **4**, 231 (1960).

⁸ Ericksen, J. L., *Trans. Soc. Rheol.*, **4**, 29 (1960).

⁹ Joel, N., and Wooster, W. A., *Acta Crist.*, **13**, 516 (1960).

¹⁰ Jaffe, H., and Smith, C. S., *Phys. Rev.*, **121**, 1604 (1961).

APPLICATION OF PALÆOMAGNETISM TO GEOLOGICAL CORRELATION

By A. LAROCHELLE

Geological Survey of Canada

THE Monteregian Hills are formed from the erosional remnants of eight igneous plugs which intrude the Palæozoic sedimentary beds of south-eastern Quebec (Fig. 1). Work on these intrusive rocks¹ by a number of geologists has shown that they

were derived from a common magmatic source of basic composition.

Suggestions have been made² to include in the Monteregian Hills series a number of alkaline intrusives which are located outside the immediate area