L*v'(t) + (v(t)^2)/2 = g*(a*exp(b*t)) , v(0)=0

Input:

$$\left\{L\,v'(t) + \frac{v(t)^2}{2} = g\left(a \exp(b\,t)\right),\; v(0) = 0\right\}$$

Riccati's equation:

$$v'(t) = -\frac{v(t)^2}{2\,L} + \frac{a\,e^{b\,t}\,g}{L}$$

Riccati's equation »

ODE classification:

first-order nonlinear ordinary differential equation

Alternate forms:

$$\left\{2\,a\,g\,e^{b\,t} = 2\,L\,v'(t) + v(t)^2,\; v(0) = 0\right\}$$

$$\left\{L\,v'(t) = a\,e^{b\,t}\,g - \frac{v(t)^2}{2},\; v(0) = 0\right\}$$

$$\left\{\frac{1}{2}\left(2\,L\,v'(t) + v(t)^2\right) = a\,e^{b\,t}\,g,\; v(0) = 0\right\}$$

Differential equation solution:

$$v(t) = \frac{\sqrt{2}\,\sqrt{a\,g\,e^{b\,t}}\left(K_1\left(\frac{\sqrt{2}\,\sqrt{a\,g}}{b\,L}\right) I_1\left(\frac{\sqrt{2}\,\sqrt{a\,e^{b\,t}\,g}}{b\,L}\right) - I_1\left(\frac{\sqrt{2}\,\sqrt{a\,g}}{b\,L}\right) K_1\left(\frac{\sqrt{2}\,\sqrt{a\,e^{b\,t}\,g}}{b\,L}\right)\right)}{I_1\left(\frac{\sqrt{2}\,\sqrt{a\,g}}{b\,L}\right) K_0\left(\frac{\sqrt{2}\,\sqrt{a\,e^{b\,t}\,g}}{b\,L}\right) + K_1\left(\frac{\sqrt{2}\,\sqrt{a\,g}}{b\,L}\right) I_0\left(\frac{\sqrt{2}\,\sqrt{a\,e^{b\,t}\,g}}{b\,L}\right)}$$

I_n(z) is the modified Bessel function of the first kind »

K_n(x) is the modified Bessel function of the second kind »